Unified diffusion tensor estimation using space-varying coefficients models

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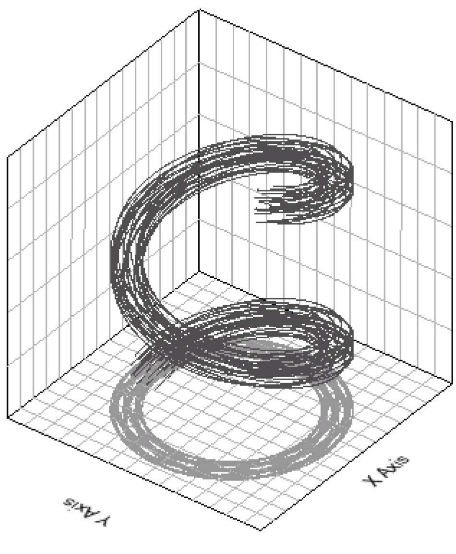
Introduction: In diffusion tensor (DT) imaging, signals are collected on a regular though artifical grid. The continuity of the tensor field is however prerequisite for any high-resolution analysis. Presently, most fiber tracking algorithms require a 3-step procedure comprising voxelwise tensor estimation (*level1*), regularization (*level2*) and interpolation of the tensor field (*level3*). We present a unified approach [1], that subsumes all processing steps simultaneously, and validate this concept compared with standard procedures (*level1* to *level3*) and different acquisition schemes (1 [repitition] x 6 [directions], 3x6, 1x15, 1x31).

Theory and Simulation Model: Currently (non-)linear regression is applied voxelwise; the six unique elements of the DT correspond to six coefficients in the regression model. The unified approach [1] combines the $N=N_1xN_2xN_3$ separate regression models of the standard approach to a joint space-varying coefficient model (SVCM) by a suitable (spatial) design of the 3d coefficient surfaces. This way, correlation among adjacent voxels is incorporated. The elements of each coefficient surface are modelled non-parametrically by projecting them onto penalized tensor product B-splines. For each regressor, difference penalties are placed on the rows, columns and layers of the tensor product coefficients, thereby forcing the coefficients to change smoothly in space. As consequence, the objective function consists of the least-squares term and a penalty term that serves to avoid over-fitting. The trade-off between smoothness and fidelity to the data is determined by a tuning parameter which is optimized by generalized cross-validation.
Here, we focus on the computationally efficient subsequent smoothing variant of the SVCM which in principle is applicable to real brain images.
For simulation, a spiral serves as simplified fiber bundle and is designed anisotropic ($□_1:□_2:□_3=2:1:1$) in fully isotropic background.
After discretization into 15x15x5 voxels, 100 artifically noised data sets were generated (□=10). To allow detail preservation, first order difference penalties are used in the SVCM estimation, either combined with linear ($SVCM_{lin}$) or quadratic ($SVCM_{quad}$) basis functions, and evaluated at original and double resolution. A global tuning parameter was used. The estimation quality was quantified the MSE averaged over all voxels, only fiber and non-fiber voxels.



Results and Discussion:
<u>Original resolution</u>: Independent of the aquisition scheme, both $SVCM_{lin}$ and $SVCM_{quad}$ outperform standard *level2* but not *level1*. This benefit is most distinct in anisotropic voxels.

Double resolution: In anisotropic conditions, also both *SVCM*$_{lin}$ and *SVCM*$_{quad}$ are superior to *level3*, however, both visual inspection and MSE analyses show inhomogeneously reproduced background. Neither increasing directions nor repeats could improve estimation quality of *SVCM*$_{quad}$. Interpolation using *SVCM*$_{lin}$ proved more accurate than tri-linearly interpolated *level1* or *level3*, also for 31 directions.

In general, SVCM variants still suffer from Gibbs phenomena and insufficent local adaptiveness. Promising developments which involve data-driven penalty weights or the use of geometrically more flexible basis functions are needed to render the SVCM applicable for fiber tracking and high-resolution analyses.

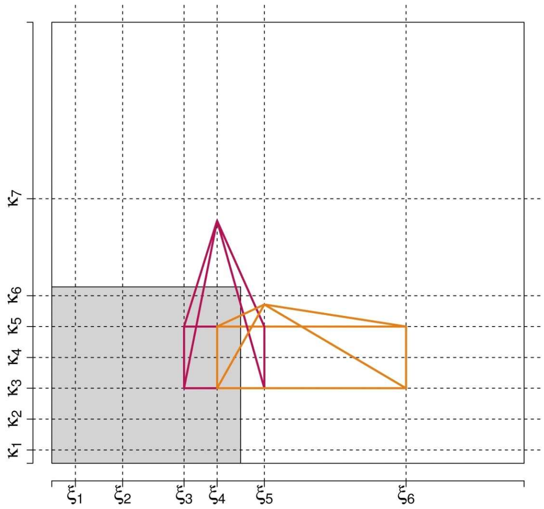


Figure 2. 2d-example exhibiting sharp edges between two regions. The amplitudes of two neighbouring tensor product B-splines could be allowed to differ appropriately by relaxing the corresponding penalty (see orange- and red-colored basis functions).

[1] Heim etal. Space-Varying Coefficient Models for Brain Imaging, *submitted*

Category: *Modeling and Analysis*